A Priori Precision Design for Estimating Gain–Probability in Nonparametric Settings

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Abstract

Gain–probability (G–P) gives an interpretable probability-based framework for comparing outcomes between two conditions. Instead of focusing on null-hypothesis rejection, G–P directly quantifies the probability that one outcome exceeds another by a specified difference or multiplicative factor. This paper develops a priori precision procedures (APP) for estimating G–P in nonparametric settings.

For paired observations, the estimator is a mean of Bernoulli indicators, which allows exact binomial calculations, normal approximations, and a fully distribution-free Hoeffding bound. For independent samples, the natural estimator averages indicator functions over all cross-pairs. Because cross-pairs reuse observations, the terms are dependent and cannot be treated as mn independent Bernoulli trials. Using McDiarmid's bounded-difference inequality, we derive a distribution-free APP rule governed by the effective sample size $n_{\text{eff}} = mn/(m+n)$.

We further strengthen the theory by giving a two-sample U-statistic representation, a variance decomposition, and an asymptotic normality result. The project also includes a detailed simulation design, reproducible `R` code, and benchmark graphics. The resulting framework provides both finite-sample and asymptotic guidance for study planning when gain–probabilities are the inferential targets.